Probabilistic Tabled Logic Programming with Application to Model Checking

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Executable Specification of Operational Semantics

$$\frac{e_1 \rightarrow e_1'}{(e_1\ e_2) \rightarrow (e_1'\ e_2)}$$

```
step(app(E1, E2), app(E1P, E2)) :-
    step(E1, E1P).
```

$$\frac{e_2 \rightarrow e_2'}{(v_1\ e_2) \rightarrow (v_1\ e_2')}$$

```
step(app(V1, E2), app(V1, E2P)) :-
    isValue(V1),
    step(E2, E2P).
```

$$(\lambda x.\ e_1)\ \ v_2 \rightarrow [x \mapsto v_2]e_1$$

```
step(app(lambda(X, E1), V2), E2) :-
    isValue(V2),
    subst(X, V2, E1, E2).

isValue(lambda(_, _)).
```

[Call-By-Value Lambda Calculus]

Substitution

$$\begin{array}{lcll}
[x \mapsto s]x & = & s & \\
[x \mapsto s]y & = & y & \text{if } y \neq x \\
[x \mapsto s](\lambda y.\ t) & = & \lambda y.\ [x \mapsto s]t & \text{if } x \neq y \text{ and } y \notin fv(s) \\
[x \mapsto s](t_1\ t_2) & = & ([x \mapsto s]t_1)\ ([x \mapsto s]t_2) &
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- This definition becomes complete only when we consider α-renaming.
- We can program α-renaming explicitly, or better still...
- With suitable restrictions on the way λ-terms are written,
  - represent variables in lambda-terms with logical variables, and
  - use the "standardization" done by resolution to perform the needed α-renaming.
- We used such a strategy to encode model checkers for the *pi*-calculus [Yang et al, VMCAI'03].

Executable Specification of Abstract Semantics

$$\frac{\texttt{p = \&q}}{\texttt{p} \rightarrow \texttt{q}}$$

```
pts(P,Q) :-
    stmt(v(P), addr(Q)).
```

$$\frac{\texttt{p = q} \quad \texttt{q} \rightarrow r}{\texttt{p} \rightarrow r}$$

```
pts(P,R) :-
    stmt(v(P), v(Q)),
    pts(Q, R).
```

$$\frac{\texttt{p = *q} \quad \texttt{q} \rightarrow r \quad r \rightarrow s}{\texttt{p} \rightarrow s}$$

```
pts(P,S) :-
    stmt(v(P), star(Q)),
    pts(Q, R), pts(R, S).
```

$$\frac{\texttt{*p = q} \quad \texttt{p} \rightarrow r \quad \texttt{q} \rightarrow s}{r \rightarrow s}$$

```
pts(R, S) :-
    stmt(star(P), v(Q)),
    pts(P, R),
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```

[Anderson's Context-Insensitive Points-To Analysis]

Demand-Driven Analysis

Compute only the information necessary to determine the *may-point-to* set of x. [Heinze et al., PLDI 2001]

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- Tabled query evaluation is naturally demand-driven, but ...
- Clauses of the form `pts(R, S) :- stmt(star(P), v(Q)), ...` lead to generate-and-test evaluation.
- **Trick:** replicate *points-to* (`pts`) as *pointed-to-by* (`ptb`).

```
pts(R, S) :-
    stmt(star(P), v(Q)),
    pts(P, R),
    pts(Q, S).
```

$\Rightarrow$

```
pts(R, S) :-
    ptb(R, P),
    stmt(star(P), v(Q)),
    pts(Q, S).
```

[PPDP'05]

Incremental Evaluation

- Computing changes to query answers for definite programs when rules/facts are *added* is relatively easy.
  - Semi-naive and tabling are naturally incremental w.r.t. addition of clauses.
- Computing changes when clauses are *deleted* is harder:
  - DRed [Gupta et al, SIGMOD'93], and similar algorithms in model checking [Sokolsky & Smolka, CAV'94] and program analysis [e.g., Yur et al, ICSE'99] have been proposed for this problem.
  - DRed is prohibitively expensive in practice.

Incremental Evaluation (contd.)

- Use of Support Graphs, to store dependency between query answers and clauses/facts, makes DRed feasible [Saha & R., ICLP'03].
- Application to *incremental program analysis* [Saha & R. PPDP'05]
- *Symbolic* support graphs significantly reduce memory requirements for certain classes of programs [Saha & R., ICLP'05].
- Subsequent generalization to handle updates [ICLP'06], and Prolog [PADL'06]

Executable Specification of Semantic Equations

$[\![\cdot]\!]$ is the smallest set such that:

% $[\![p]\!]$ = states satisfying prop. p.
$[\![p]\!] = \{s \mid p \in AP(s)\}$

% Conjunction:
$[\![\varphi_1 \wedge \varphi_2]\!] = [\![\varphi_1]\!] \cap [\![\varphi_2]\!]$

% $[\![\text{EF } f]\!] =$
% $\quad \{s \mid \exists t.\ s \xrightarrow{*} t \text{ and } t \in [\![f]\!]\}$
$[\![EF\varphi]\!] = [\![\varphi]\!]$
$\quad \cup \{s \mid \exists t.\ s \rightarrow t, t \in [\![EF\varphi]\!]\}$

⋮

```
models(S,prop(P)) :-
    holds(S, P).

models(S,and(F1,F2)) :-
    models(S, F1), models(S, F2).

models(S, ef(F)) :-
    models(S, F).
models(S, ef(F)) :-
    trans(S, T), models(T, ef(F)).

models(S, af(F)) :-
    models(S, F).
models(S, af(F)) :-
    findall(T, trans(S, T), L),
    all_models(T, af(F)).
...
```

[Computation Tree Logic's Semantics (Fragment)]

Model Checking and Program Analysis as Query Evaluation

Mobile Ad-Hoc Networks

Parameterized Systems

Multi-Agent Systems

Model Checkers

`Infinite-State Systems`

π-Calculus

Incremental Program Analyzers

Program Analyzers

`Alias Analysis of C Programs`

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Other Analyzers

Security Policy Analyzers

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Logic Programs

Program Rules

$+$ $\models$ **Query Answers**

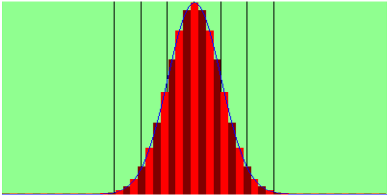
Facts

Probabilistic Logic Programs

Program Rules

$+$ $\models$ **Query Answers**

Probabilistic Facts



The PRISM language and system [Sato and Kameya '97]

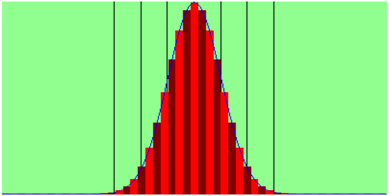
Probabilistic Logic Programs

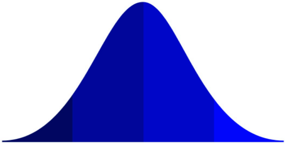
Program Rules

$+$

$\models$ **Query Answers**

Probabilistic Facts





The PRISM language and system [Sato and Kameya '97]

PRISM

A language for probabilistic logic programming with system for inference and parameter learning (Sato et al, since '99).

- Logic programs with a set of **probabilistic facts**: `msw(X, I, V)`, where
  - `X` is a discrete-valued random process
  - `V` is a value generated by the random process
  - `I` is the *instance number*, distinguishing different trials.
- Random variables generated by the same random process are i.i.d.
- Random variables generated by distinct random processes are independent.
- Has a well-defined model-theoretic (*distribution*) semantics, and an operational semantics based on tabled resolution.

Distribution semantics

```
% "a" is a boolean random process
p(X) :-  msw(a, 0, X),
         msw(a, 1, Y),
         X=Y.
values(a, [t,f]).
set_sw(a, [0.3,0.7])
```

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Worlds:

msw(a,0,t) msw(a,1,t)	msw(a,0,t) msw(a,1,f)
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- Outcomes of random processes define worlds.

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Worlds:

msw(a,0,t) msw(a,1,t) 0.09	msw(a,0,t) msw(a,1,f) 0.21
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- In each world, `msw`s form a set of logical (non-probabilistic) facts.

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Models:

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- Outcomes of random processes define worlds.
- The probability of a world is assigned based on the probabilities of the outcomes in the world.
- In each world, `msw`s form a set of logical (non-probabilistic) facts.
- Distribution over least models: the least model in each world is assigned the probability of that world.

Probabilistic Logic Programs: Background

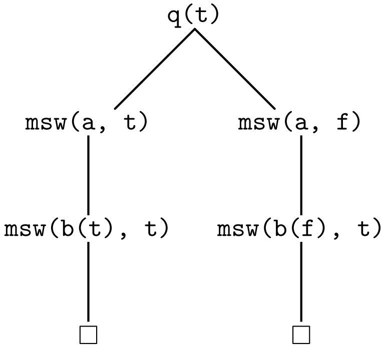
- Logic-based representation of statistical models
  - Examples include BLPs (Kersting and De Raedt, '00), PRMs (Friedman et al, '99), MLNs (Richarson and Domingos, '06).
  - The underlying statistical network, derived from logical/statistical specifications, is finite.
- Statistical inference over proof structures
  - Conservative extension to traditional logic programs, with explicit or implicit use of random variables and processes.
  - Examples include PRISM (Sato and Kameya, '99), ICL (Poole, '93), CLP(BN) (Santos Costa et al, '03), ProbLog (De Raedt et al, '07), LPAD (Vennekens et al, '09).
  - In terms of expressive power, PRISM, ProbLog and LPAD coincide; however, they use different inference procedures.

Evaluation in PRISM — I

```
% Finite Mixture Model
q(Y) :- msw(a, 0, X),
        msw(b(X), 0, Y).

values(a, [t,f]).
values(b(_), [t,f]).
set_sw(a, [0.3,0.7])
set_sw(b(t), [0.6,0.4])
set_sw(b(f), [0.5,0.5])
```

Explanations



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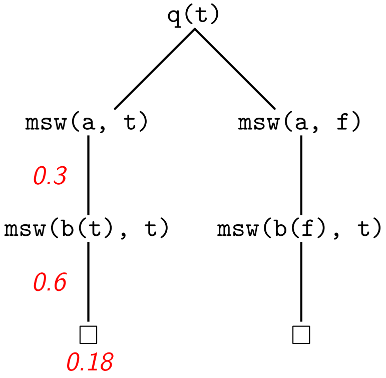
Explanations *and Probabilities*



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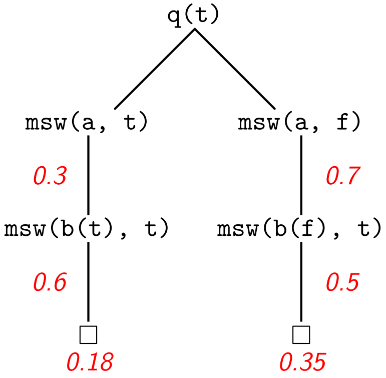

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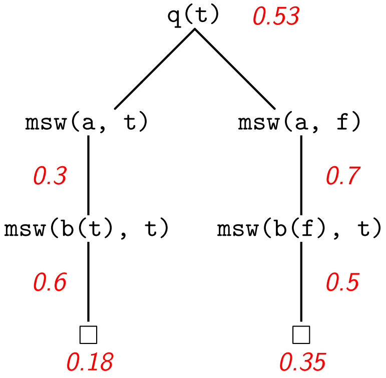

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  - If explanations are pairwise mutually exclusive, then the probability of the set of explanations is the sum of probabilities of each explanation.
    [Mutual Exclusion assumption]
  - If the set of explanations is finite, then this sum can be effectively computed.
    [Finiteness assumption]

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- ProbLog and PITA (an implementation of LPAD) use BDDs to represent the set of explanations, and consequently remove Independence and Mutual Exclusion assumptions.
  - Finiteness assumption is still needed since the BDDs need to be effectively constructed.

Probabilistic Systems

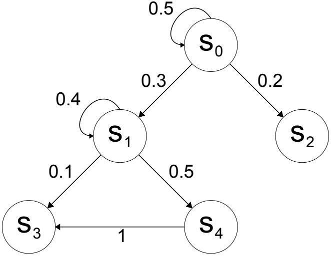
- **System Definitions:** Markov Chains (discrete- and continuous-time), Markov Decision Processes, Probabilistic Automata, recursive versions of some of the above, ...
- **Property Specifications:** PCTL, PCTL*, CSL, GPL, ...
- **Systems:** Prism, PreMo, UPPAAL-SMC, ...

Systems have stochastic behavior

... in contrast to *Statistical Model Checking* where statistical (sampling) techniques are used to infer properties of non-probabilistic systems (with confidence bounds).

Probabilistic Transition Systems in PRISM

Example Markov Chain



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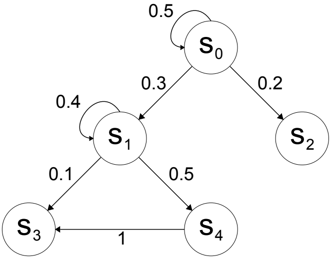


```
% Encoding as a Probabilistic LP
trans(S, I, T) :- msw(t(S), I, T).

% Ranges
:- values(t(s0), [s0, s1, s2]).
:- values(t(s1), [s1, s3, s4]).
:- values(t(s4), [s3]).

% Distributions
set_sw(t(s0), [0.5, 0.3, 0.2]).
set_sw(t(s1), [0.4, 0.1, 0.5]).
set_sw(t(s4), [1]).
```

Probabilistic Transition Systems in PRISM

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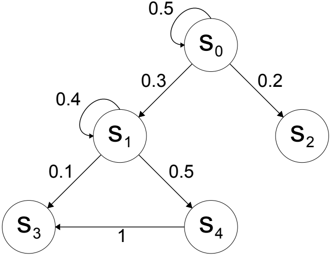
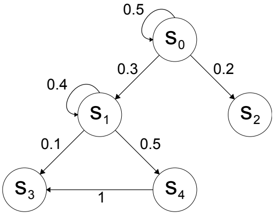



```
% Encoding as a Probabilistic LP
trans(S, I, T) :- msw(t(S), I, T).

% Encoding of Reachability
reach(S, I, T) :-
    trans(S, I, U),
    reach(U, next(I), T).
reach(S, _, S).
```

Probabilistic Model Checking as Query Evaluation

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- What is the probability of reaching s_3 via some path starting at s_0?

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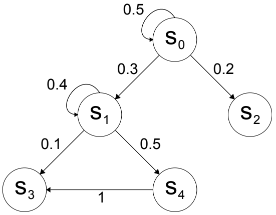


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- `|?- prob(reach(`s_0`, 0, `s_3`)).`

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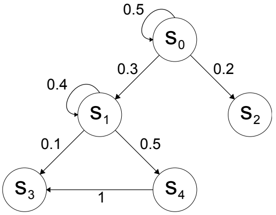


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reach(S, _, S).
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- What is the probability of reaching s_3 via some path starting at s_0?
- `|?- prob(reach(s0, 0, s3)).`
- Evaluation of the above query will not terminate!

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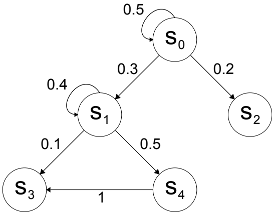


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- What is the probability of reaching s_3 via some path starting at s_0?
- `|?- prob(reach(`s_0`, 0, `s_3`)).`
- Evaluation of the above query will not terminate!
  - There are infinitely many *explanations* for `reach(`s_0`, 0, `s_3`)`

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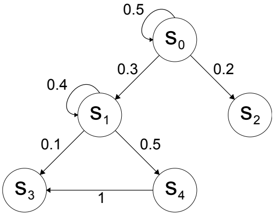


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- Evaluation of the above query will not terminate!
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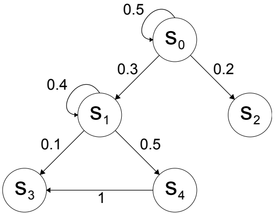
Probabilistic Model Checking as Query Evaluation




```
trans(S, I, T) :-
    msw(t(S), I, T).

reach(S, I, T) :-
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reach(S, _, S).
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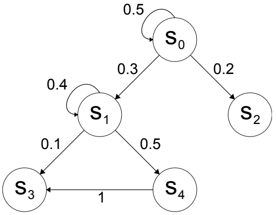


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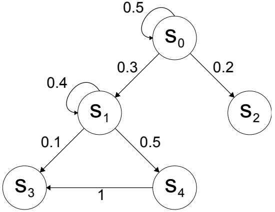
- Distribution semantics is well-defined and gives the correct probability, but
  - PRISM/ProbLog/PITA cannot evaluate this query.
- "PIP" solves this problem [Gorlin, R. & Smolka, ICLP'12].

Explanations

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```

Explanations for `reach(s0,0,s3)`:

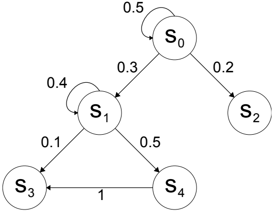
- `msw`(t(s0), 0, s1), `msw`(t(s1), *next*(0), s3).
- `msw`(t(s0), 0, s0), `msw`(t(s0), *next*(0), s1), `msw`(t(s1), *next*(*next*(0)), s3).

⋮

- `msw`(t(s0), 0, s1), `msw`(t(s1), *next*(0), s1), `msw`(t(s1), *next*(*next*(0)), s3).

⋮

Explanations

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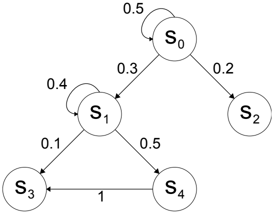
Note: `prob(reach(s0,0,s3))` is same as `prob(reach(s0,H,s3))` for any H.

Explanations

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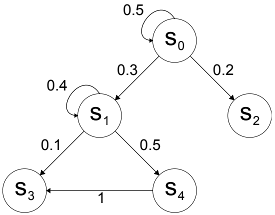


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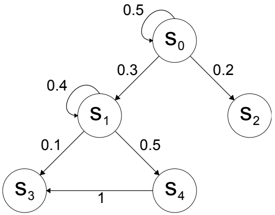
Note: `prob(reach(s0,0,s3))` is same as `prob(reach(s0,H,s3))` for any `H`.

We can use a *grammar* to represent the set of explanations for the abstracted query.

$$\texttt{expl}(\texttt{reach}(s0, H, s3)) \longrightarrow [\texttt{msw}(t(s0), H, s0)],\ \texttt{expl}(\texttt{reach}(s0, \mathit{next}(H), s3)).$$

$$\texttt{expl}(\texttt{reach}(s0, H, s3)) \longrightarrow [\texttt{msw}(t(s0), H, s1)],\ \texttt{expl}(\texttt{reach}(s1, \mathit{next}(H), s3)).$$

Explanations

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$\mathtt{expl}(\mathtt{reach}(s0, H, s3)) \longrightarrow$
$[\mathtt{msw}(t(s0), H, s0)],$
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$\mathtt{expl}(\mathtt{reach}(s0, H, s3)) \longrightarrow$
$[\mathtt{msw}(t(s0), H, s1)],$
$\mathtt{expl}(\mathtt{reach}(s1, next(H), s3)).$

is similar to the stochastic grammar:

$S_0 \xrightarrow{0.5} S_0$
$S_0 \xrightarrow{0.3} S_1$

whose probability is given by the least solution to the equation:
$x_0 = 0.5x_0 + 0.3x_1$

Temporally Well-Formed Programs

- A probabilistic logic program with annotations of the form `temporal`$(p/n - i)$.
  - Example: `temporal(reach/3-2)`
  - `reach` is a *temporal* predicate
  - The second argument of an atom with root `reach` is its *instance argument*.
- For a rule defining a temporal predicate, the instance argument of the head must be a subterm of instance arguments of every temporal body predicate.
  - Example:
    ```
    reach(S, I, T) :-
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    ```
- Instance arguments are not bound to non-instance arguments, or vice versa.

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```
reach(S, I, T) :-
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```
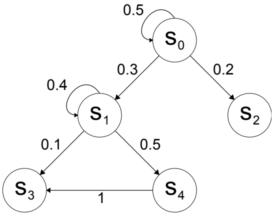

- Instance arguments are not bound to non-instance arguments, or vice versa.
- In explanation grammars of temporally well-formed programs, `msw`(r, t, x) will always be independent of any `msw` derived from non-terminal `expl(p)`
  - if t is a proper subterm of p's instance argument.

Factored Equation Diagrams

Not all explanation grammars can be translated directly to stochastic grammars.



- Consider the explanation grammar for query

  `reach(s0, H, s3); reach(s0, H, s4).`

- The grammar will have productions of the form:

  $\texttt{expl}(\texttt{reach}(s0, H, s3); \texttt{reach}(s0, H, s4)) \longrightarrow \texttt{expl}(\texttt{reach}(s0, H, s3)).$

  $\texttt{expl}(\texttt{reach}(s0, H, s3); \texttt{reach}(s0, H, s4)) \longrightarrow \texttt{expl}(\texttt{reach}(s0, H, s4)).$

We can *factor* such grammars using Factored Explanation Diagrams (FEDs), which are similar to BDDs.

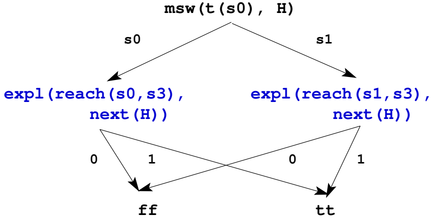
Structure of FEDs

FED is a labeled DAG with

- `tt` and `ff` as leaf nodes
- `msw`(r, h) is an n-ary node if r is a random process with n possible outcomes;

  outgoing edges are labeled with the outcomes.
- `expl`(t, h) is a binary node;

  outgoing edges are labeled 0 and 1.
- If there is an edge from x_1 to x_2, then $x_1 < x_2$ via a specially defined partial order relation.



Operations on FEDs

Boolean operations "$\wedge$" and "$\vee$" can be performed on FEDs along the same line as on BDDs, with one significant change:

- BDD operations are based on a *total* node order.
- We only have a partial node order for FEDs.
- When we recursively push operations down the diagram, we may encounter incomparable nodes.
- We then generate a placeholder `merge` node, and process merges separately.

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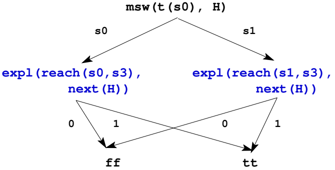
- We only have a partial node order for FEDs.
- When we recursively push operations down the diagram, we may encounter incomparable nodes.
- We then generate a placeholder `merge` node, and process merges separately.
- Note that `msw` nodes are always comparable; so a `merge` will involve at least one `expl` node.
- We expand (one of) the `expl` node(s) with its definition, and perform the postponed operation.

FEDs to Equations

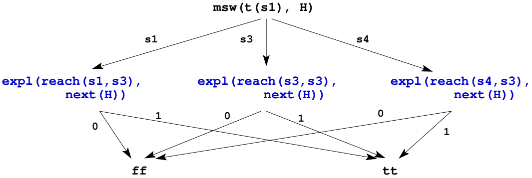
The probability of a set of explanations is computed by generating and solving a set of equations from its FED.

FED for `expl(reach(s0,s3), H)`:



$$x_0 = t_{00} * x_0 + t_{01} * x_1$$
$$t_{00} = 0.5$$
$$t_{01} = 0.3$$

FED for `expl(reach(s1,s3), H)`:



$$x_1 = t_{11} * x_1 + t_{13} * x_3 + t_{14} * x_4$$
$$t_{11} = 0.4$$
$$t_{13} = 0.1$$
$$t_{14} = 0.5$$

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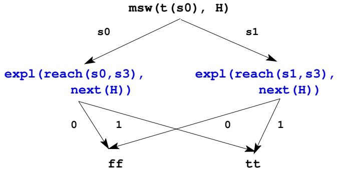
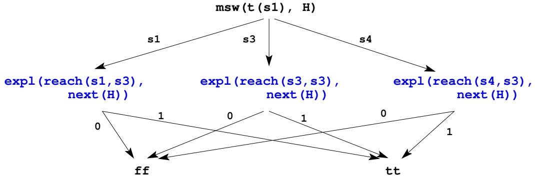


$$
\begin{aligned}
x_0 &= t_{00} * x_0 \\
&\quad + t_{01} * x_1 \\
t_{00} &= 0.5 \\
t_{01} &= 0.3
\end{aligned}
$$

FED for `expl(reach(s1,s3), H)`:



$$
\begin{aligned}
x_1 &= t_{11} * x_1 \\
&\quad + t_{13} * x_3 \\
&\quad + t_{14} * x_4 \\
t_{11} &= 0.4 \\
t_{13} &= 0.1 \\
t_{14} &= 0.5
\end{aligned}
$$

The least solution to these monotone polynomial equations gives the probability of the set of explanations.

Probabilistic Computation Tree Logic (PCTL)

- PCTL is a logic for specifying properties of Probabilistic Transition Systems (Discrete-Time Markov Chains), where a subset of predefined *propositions*, A, hold at states.
- State formulas, φ, defined over individual states:

$$A \quad | \quad \neg\varphi \quad | \quad \varphi_1 \wedge \varphi_2 \quad | \quad \varphi_1 \vee \varphi_2 \quad | \quad Pr(\psi) > b \quad | \quad Pr(\psi) \geq b$$

- Path formulas, ψ, defined over execution paths:

$$\phi_1 \; U \; \phi_2 \quad | \quad X \; \phi$$

- State formulas are non-probabilistic; path formulas have associated probabilities.
- Used as the property specification language by many systems, including the Prism Model Checker.

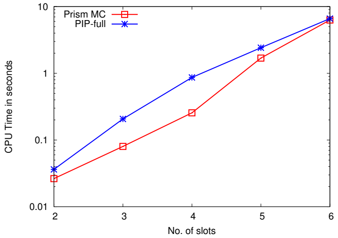
Encoding the PCTL Model Checker

```
% State Formulae
models(S, prop(A)) :-
    holds(S, A).
models(S, neg(SF)) :-
    not models(S, SF).
models(S, and(SF1, SF2)) :-
    models(S, SF1),
    models(S, SF2).
models(S, pr(PF, gt, B)) :-
    prob(pmodels(S, PF), P),
    P > B.
models(S, pr(PF, geq, B)) :-
    prob(pmodels(S, PF), P),
    P >= B.
```

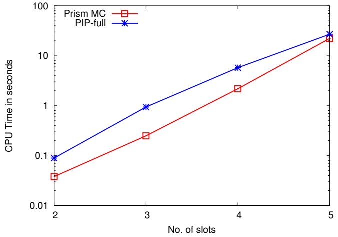
```
% Path Formulae
pmodels(S, PF) :-
    pmodels(S, PF, _).

:- table pmodels/3.
pmodels(S, until(SF1, SF2), H) :-
    models(S, SF2).
pmodels(S, until(SF1, SF2), H) :-
    models(S, SF1),
    trans(S, H, T),
    pmodels(T, until(SF1, SF2), next(H)).
pmodels(S, next(SF), H) :-
    trans(S, H, T),
    models(T, SF).

temporal(pmodels/3-3).
```

Prototype: PCTL Model Checking

5 processes:



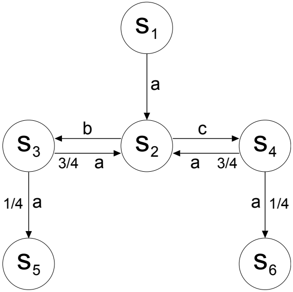
6 processes:



- Time performance is compared with that of the Prism Model Checker.
- System specified using Prism's modeling language (Reactive Modues, RM).

  Markov Chain derived from direct logical encoding of the semantics of RM.
- Chosen benchmark:
  - *System*: Synchronous Leader Election protocol
  - *Property*: "eventually a leader is elected" (reachability).
- Model checking times are within a factor of 3 (note log scale).

Reactive Probabilistic Labeled Transition Systems (RPLTS)



- Automata has finite number of states.
- Each state offers a finite number of *actions*, each with a distinct label.
- Each action has a *distribution* of states: taking an action chooses a destination state according to the given distribution.
- Actions are triggered by an external agent; the system *reacts* to actions.

[Cleaveland, Iyer & Narasimha, TCS'05]

Generalized Probabilistic Logic (GPL)

[Cleaveland, Iyer & Narasimha, TCS'05]

- An expressive, mu-calculus-based, logic for branching-time probabilistic processes.
- Strictly more expressive than PCTL*.
- Can be used to construct model checkers for recursive Markov Chains.
- *Thus far, no model checker was available!!*
- We can construct a model checker for GPL by directly encoding its semantics as a probabilistic logic program.

GPL

- Usual mu-calculus-like modalities and fixed points (called “state formulae”) in GPL.
- Fuzzy formulae, ψ, have a probabilistic interpretation: each formula's truth value has a probability associated with it.

$$\psi = \psi \vee \psi \mid \psi \wedge \psi \mid \langle a \rangle \psi \mid [a]\psi \mid \phi \mid X$$

- State formulae, ϕ, have a boolean interpretation:

$$\phi = \phi \vee \phi \mid \cdots \mid \mathtt{pr}^{>B}\psi \mid \mathtt{pr}^{\geq B}\psi \mid \cdots \text{ propositions} \ldots$$

- Alternation-free fixed point equations of the form $X =_\mu \psi$ and $X =_\nu \psi$.

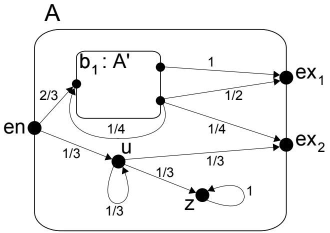
GPL Model Checker

%% pmodels(S, PF, H): S is in the model of fuzzy formula PF at or after instant H
%% smodels(S, SF): S is in the model of state formula SF

```
pmodels(S, sf(SF), H) :-
    smodels(S, SF).
pmodels(S, and(F1,F2), H) :-
    pmodels(S, F1, H),
    pmodels(S, F2, H).
pmodels(S, or(F1,F2), H) :-
    pmodels(S, F1, H);
    pmodels(S, F2, H).
pmodels(S, diam(A, F), H) :-
    action(S, A, SW),
    msw(SW, H, T),
    pmodels(T, F, [T,SW|H]).
pmodels(S, box(A, F), H) :-
    findall(SW, action(S,A,SW), L),
    all_pmodels(L, S, F, H).

pmodels(S, form(X), H) :-
    tabled_pmodels(S, X, H1), H=H1.

all_pmodels([], _, _, _H).
all_pmodels([SW|Rest], S, F, H) :-
    msw(SW, H, T),
    pmodels(T,F,[T,SW|H]),
    all_pmodels(Rest, S, F, H).

:- table tabled_pmodels/3.
tabled_pmodels(S,X,H) :-
    fdef(X, lfp(F)),
    pmodels(S, F, H).
```

Recursive Markov Chains (RMCs)

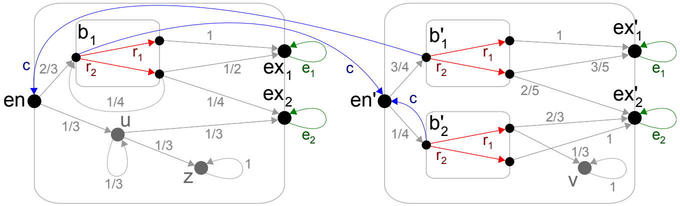
Markov chains with *calls* and *returns* [Etessami & Yannakakis, 2005, ...]



- Probabilistic Push-Down Systems [Kucera, Esparza & Mayr, 2006]
- PreMo system [Wojtczak & Etessami, 2008]

Reachability in RMCs

Transform into a Reactive Probabilistic LTS:



- Labels on probabilistic transitions are all p (omitted in figure).
- Check reachability using the following GPL formula:

X_i: eventually exit ex_i is reached:

$$
\begin{array}{rcl}
X_i & =_\mu & \langle e_i \rangle \texttt{tt} \quad \vee \quad \langle p \rangle X_i \\
 & & \vee\ (\langle c \rangle X_1 \ \wedge\ \langle r_1 \rangle X_i) \\
 & & \vee\ (\langle c \rangle X_2 \ \wedge\ \langle r_2 \rangle X_i)
\end{array}
$$

Markov Decision Processes (MDPs)

- MDP looks very similar to an RPLTS: actions on states that have a distribution of destination states.
- Semantics is different in two ways:
  - States have "*rewards*", and induce rewards on paths.
  - Schedulers dictate actions taken at each state.
- Interesting problem: find an *optimal* scheduler that maximizes the expected reward.

Committed Choice

- A scheduler commits an MDP to take a specific action at some point in its run.
- Analogous to `msw` in PRISM, we introduce `nd(X, I, V)` to choose from a set and commit to that choice.
  - `X` is a discrete-valued choice process
  - `V` is a value generated by the choice process
  - `I` is the *instance number*.
- Example: `nd`(s_2`, 0, X)` with `values`(s_2`, [b,c])` will `X` to `b` in one set of worlds, and to `c` in another.
- Distribution semantics is naturally extended: the meaning of a program is a distribution of **sets of** models.

Committed Choice (contd.)

```
q(Y) :- nd(f, 0, X),
        msw(X, 0, Y).
values(f, [a,b]).
values(a, [t,f]).
values(b, [t,f]).
set_sw(a, [0.3, 0.7])
set_sw(b, [0.6, 0.4])
```

```
?-  prob(q(t), P).

P = 0.3
;
P = 0.6
```

- Probability of an answer is computed separately for each distinct set of committed choices.
- For recursive programs (MDPs), each set of committed choices will yield a set of linear equations, whose least solution will be the corresponding probability.
- Expected rewards can be computed analogously.
- We can find optimal probabilities (and, similarly, optimal expected reward) by pushing a `max` operation into the equations themselves.

Approximate Inference

Current, Preliminary Work, on MCMC-based Sampling

- Monte-Chain Monte Carlo: walk though the possible worlds.
- Gibbs sampler: walk by resampling one of the random variables in the current state.
- In our case, we consider a **set** of possible worlds as a state in the Markov Chain. Naive method:
  - Generate a sample *derivation*. Its `msws` define a set of possible worlds.
  - Choose an `msw` and resample; find a derivation consistent with the new set of possible worlds.
  - The set of `msws` in the new derivation forms the next state in the chain.
- Using explanations instead of derivations makes this method more complex ([Moldovan et al, ECSQARU'13])

Approximate Inference for Conditional Queries

- Naive method: use Metropolis-Hastings and reject samples inconsistent with evidence.
- Better methods: Adapt sampling to not generate inconsistent examples in the first place.
  - Adapt `msw` distributions to minimize generation of samples inconsistent with evidence [e.g. Mansinghka '09].
  - Adapt the Markov Chain based on prior rejections to focus on consistent part of the state space [classical adaptive MCMC].

Current and Future Work

Sampling-Based Inference

Structure Learning (ILP)

Different Forms of Uncertainty

Expectations

"Stratification"

?

Decision Support / Planning

Statistical Model Checking

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